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Parton interaction in super Yang Mills theory

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Abstract: We apply the effective action scheme to the leading parton interactions in $\mathcal{N} = 1$ supersymmetric gauge theory. The effective interaction in the Bjorken asymptotics at one loop is written in terms of parton superfield vertices explicitely symmetric with respect to superconformal transformations.

Keywords: Conformal and W Symmetry, QCD, Deep Inelastic Scattering.

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Contents

1. Introduction

High-energy processes related to the Bjorken asymptotics [1, 2] are playing a major role in the investigation of hadronic structure and interaction.

Aspects of conformal symmetry played an important role in the development of the first concepts about the Bjorken limit and turned into an useful tool, e.g. for finding multiplicatively renormalized operators [3] and for relating the forward to the non-forward evolution [4, 5]. In combination with supersymmetry it allows to derive intersting relations between the QCD DGLAP/ERBL evolution kernels [4]. A recent review is given in [6].

Scattering amplitudes in the Bjorken asymptotics and the renormalization of composite operators contributing to this asymptotic expansion have been studied in QCD and in many field theory models in particular for developing the methods of calculation and for investigating the symmetry properties in the asymptotics in comparison with the symmetries of the underlying theory.

A particular feature of symmetry encountered in some cases and approximations is integrability, which attracted much interest in the last years. It has been first noticed by Lipatov [8] in studying the perturbative Regge asymptotics of QCD [7] and formulated in further detail by Faddeev and Korchemsky [9]. The methods of integrable systems have been developed for the needs of such applications (see e.g. [10, 11]) and applied in particular to the renormalization of higher twist composite operators, e.g. [12, 13]. Questions related to the AdS/CFT hypothesis motivated a series of studies of special composite operators in $\mathcal{N} = 4$ super Yang- Mills theory, where also integrable structures have been encountered [14, 15].

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There is a remarkable relation between the Regge and the Bjorken asymptotics in $\mathcal{N} = 4$ super Yang-Mills theory: The eigenvalues of the Bjorken evolution kernels, the twist-2 anomalous dimensions, can be obtained from the eigenvalues of the Regge BFKL kernel by continuation to particular values of the conformal weight [17].

These developments motivated recent papers on the Bjorken asymptotics in theories with supersymmetry [18, 19, 20] and provide also a motivation for the present study.

In a recent paper [20] Belitsky et al. have treated the supersymmetric $\mathcal{N} = 1, 2, 4$ supersymmetric Yang- Mills theories emphasizing the maximally extended $\mathcal{N} = 4$ case and the relation to integrability. Their result on the two-parton interaction is restricted to the term having non-vanishing contribution in the configuration of parallel helicities, which is the only one remaining in the $\mathcal{N} = 4$ case.

In a previous study [16] we have considered the Bjorken asymptotics of QCD in the effective action approach with the aim of a close comparison to the Regge asymptotics. The effective parton interaction at one loop has been formulated in terms of vertices involving parton fields residing on the one-dimensional light ray and kernels being conformal 4-point functions. In this way we have achieved a formulation of the one-loop DGLAP/ERBL evolution where conformal symmetry is explicit.

The method of calculation and the way of symmetric formulation of the result can be applied to other models. In this paper we consider $\mathcal{N} = 1$ supersymmetric Yang-Mills theory (SYM).

In the first treatment of the ultraviolet finiteness of $\mathcal{N} = 4$ super Yang-Mills theory S. Mandelstam [21] proposed the light cone superfield formulation. This formulation has been investigated also in [22]. Our formulation deviates slightly in the way of writing the superfields: In [21, 22] $\mathcal{N}$ pairs of Grassmann variables $\theta_A, \bar{\theta}_A, A = 1, \ldots, \mathcal{N}$ are used. Here we shall use (for $\mathcal{N} = 1$) just one variable θ. In the $\bar{\theta}\theta$ formulation applied in [20] the superconformal symmetry is represented by the algebra $sl(2|\mathcal{N})$ whereas in our case for $\mathcal{N} = 1$ we are lead to consider the symmetry algebra $osp(2|1)$.

In section 2 we summarize some results on QCD of the previous paper [16]. We discuss in some detail the appropriate formulation of conformal symmetry. We present the main steps leading to the symmetric form of the effective interaction in a way that allows to demonstrate the analogy to the case of super Yang-Mills in the following.

We avoid to compose the result for the super Yang-Mills theory out of the ones for the gluon and fermion components. Instead we perform the calculation in a simple super field language in section 3. In the first step we obtain the result symmetric with respect to super translations.

In section 4 we write the generic form of the kernels of super conformal symmetric operators. We give a basis of parton wave functions and of composite parton field operators, related by a symmetric inner product, corresponding to irreducible representations of the superconformal symmetry. In this way we get the eigenstates or multiplicatively renormalized operators of the parton evolution operators. The corresponding eigenvalues are the anomalous dimensions.

In section 5 we derive the super conformal symmetric interaction vertices. This relies on the known generic form of symmetric kernels and on the analogy to the gluodynamic case.

2. Symmetry of parton interactions in QCD

2.1 Calculation of kernels

The kernels of the DGLAP/ERBL evolution can be considered as effective vertices of parton interaction $1'2' \to 12$. The partons carry merely one light-cone momentum component and can be represented by fields living on the light-ray. It is well known that this interaction at one loop level is symmetric with respect to conformal transformations acting on this light ray by Möbius substitutions (for a recent review see [6]). The kernels have been identified as conformal 4-point functions [16]. The parton fields on the light ray emerge as particular modes of the QCD gluon and quark fields by eliminating the redundant field components in light-cone gauge and by separating the high virtual modes A_p with differences of coordinates close to the light ray,

$$A(x_1)A(x_2) \to A_{p1}(z_1)A_{p2}(z_2)\delta_Q(x_{12}^{\perp})\delta_Q(x_{12+}) \tag{2.1}$$

and the low virtual modes $A_{p'}$ almost constant in a broad range vertical to the light ray,

$$A(x) \to A_{p'}(z)\Delta_m(x^{\perp}) . \tag{2.2}$$

Here δ_Q stands for narrow distribution of width $\sim Q^{-1}$ and Δ_m for a flat distribution of width $\sim m^{-1}$, $m \ll Q$.

We represent 4-vectors x^μ by their light cone components $x_\pm$ and a complex number involving the transverse components $x_\perp = x_1 + ix_2$. In the case of the gradient vector we change the notation in such a way to have $\partial_+ x_- = \partial_- x_+ = 1, \partial_\perp x = \partial_\perp^* x^* = 1$. We choose the frame where the light-like vector q' has the only non-vanishing component $q'_- = \sqrt{s/2}$ and p' the only non-vanishing component $p'_+ = \sqrt{s/2}$.

The fields $A(x)$ denote in the case of gluons the transverse components A, A^* of the vector potential in the light cone gauge $A_- = 0$, where A_+ has been integrated out. In the case of fermions they denote the light cone components $f, f^*, \tilde{f}, \tilde{f}^*$ defined as follows.

$$\begin{aligned} \psi &= \psi_- + \psi_+ , & \gamma_-\psi_+ &= \gamma_+\psi_- = 0 \\ \psi_+ &= f u_{+-} + \tilde{f} u_{++} , & \gamma^\mu &= \frac{1}{\sqrt{s}}(\gamma_- q'^\mu + \gamma_+ p'^\mu) + \gamma_\perp^\mu . \end{aligned} \tag{2.3}$$

$u_{a,b}$, $a, b = \pm$, is a basis of Majorana spinors,

$$\gamma_+ u_{-,b} = \gamma_- u_{+,b} = 0 , \qquad \gamma u_{a,-} = \gamma^* u_{a,+} = 0 . \tag{2.4}$$

The QCD interaction in terms of these component fields can be reconstructed by gauge symmetry starting from the kinetic terms

$$S_{\text{kin}} = -\int d^4x 2A^{*a}(\partial_+\partial_- - \partial^\perp\partial^{\perp *})A^a . \tag{2.5}$$

The gauge group structure will be written by using brackets combining two fields into the colour states of the adjoint (a) and of the two fundamental $(\alpha$ and $*\alpha)$ representations:

$$(A_1^* T^a A_2) = -if^{abc} A_1^{*b} \; A_2^c , \qquad (f_1^* t^a f_2) = t^a_{\alpha\beta} f_1^{*\alpha} f_2^\beta , \tag{2.6}$$

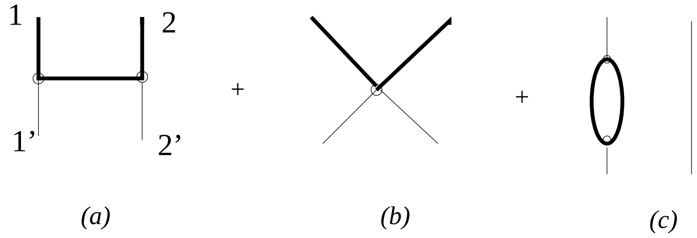

Figure 1: Graphs contributing to the one-loop effective parton interaction.

We shall restrict the detailed discussion to pure gluodynamics.

The interaction terms are recovered from the kinetic term written as

$$-2A^{*a}\partial_+\partial_- A^a - \partial^\perp \partial A^{*a}\partial^{-2}\partial^{\perp *}\partial A^a$$

by extending the transverse derivatives in this form, relying on the residual gauge symmetry,

$$\partial^\perp A^a \to (\mathcal{D}^\perp A)^a = \partial^\perp A^a + \frac{ig}{2}(A^* T^a A)\,. \tag{2.7}$$

The result can be written as

$$\begin{aligned} S &= S_{\rm kin} + S_3 + S_4\,, \\ S_3 &= \frac{g}{2}\int d^4x (\partial_1^2 \hat{V}^*_{123} A^a(x_1)(A^*(x_2)T^a A^*(x_3)|_{x_i=x} + \text{c.c.}), \\ S_4 &= \frac{g^2}{4}\int d^4x \hat{V}_{11',22'}(A^*(x_1)T^c \partial A(x_{1'})(\partial A^*(x_2)T^c A(x_{2'}))|_{x_i=x_i'=x}\,. \end{aligned} \tag{2.8}$$

The elimination of redundant field components has lead to non-local vertices,

$$\hat{V}^*_{123} = \frac{i}{3\partial_1\partial_2\partial_3}[\partial^*_{\perp 1}(\partial_2 - \partial_3) + \text{cycl.}]\,, \qquad \hat{V}_{11',22'} = (\partial_1 + \partial_{1'})^{-2}\,. \tag{2.9}$$

Here and in the following we omit the space index $+$ on derivatives, i.e. derivative operators not carrying subscripts $-, \perp$ are to be read as ∂_+. Integer number subscripts refer to the space point on which the derivative acts. The definition of the inverse ∂^{-1} is to be specified.

Formally the effective parton interaction can be obtained by substituting the field components as

$$A = A_p + A_q + A_{p'} \tag{2.10}$$

and integrating over the quantum fluctuations A_q in logarithmic approximation. In the resulting vertices involving the A_p and $A_{p'}$ modes the integration over the coordinates $x^\perp, x_+$ can be done approximately since the dependence of these field modes on them is specified. We are interested in the vertices involving two A_p and two $A_{p'}$. They result from graphs figure 1. We encounter two types of integrals and extract their logarithmic contributions. The one with 3 propagators, figure 1a, results in

$$\begin{aligned} &\int \partial_1^\perp \partial_2^{\perp *}[x_{11'}^{-2} x_{22'}^{-2} x_{1'2'}^{-2}] \to J_{111}\int \frac{d^2x^\perp}{|x^\perp|^2} \\ &J_{111} = \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(\sum \alpha_i - 1\right)\delta(z_{11'} - \alpha_1 z_{12})\delta(z_{22'} + \alpha_2 z_{12})\,. \end{aligned} \tag{2.11}$$

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The other integral involves two propagators, figure $1b$,

$$\int x_{11'}^{-2} x_{22'}^{-2} \delta^{(4)}(x_{1'2'}) \to J_0 \int \frac{d^2 x^\perp}{|x^\perp|^2}$$
$$J_0 = \int_0^1 d\alpha_1 d\alpha_2 \delta\left(\sum \alpha_i - 1\right) \delta(z_{11'} - \alpha z_{12}) \delta(z_{22'} + \alpha z_{12}) \,. \tag{2.12}$$

The first integrals in (2.11), (2.12) are over the coordinates of the points $i = 1, 2, 1', 2'$ on which the reduced parton fields $A_p, A_{p'}$ (2.1) do not depend, i.e.

$$\int \prod_i d^2 x_i^\perp dx_{i+} \, \delta_Q(x_{12}^\perp) \, \delta_Q(x_{12+}) \Delta_m(x_{1'}^\perp) \Delta(x_{2'}^\perp) \cdots \,. \tag{2.13}$$

Whereas $x_{12}^\perp, x_{12+}$ are restricted to small values the integral over $x_1^\perp, x_{1+}$ results in the condition of small momentum transfer. We have written the logarithmic contribution of the remaining integral over the transverse and + components of $x_{1'}, x_{2'}$.

The result of the disconnected contributions, figure $1c$, has the form

$$\delta(z_{11'}) \delta(z_{22'}) C_p \left(\int_0^1 \frac{d\alpha}{\alpha} + w_p \right) \cdot \int \frac{d^2 x^\perp}{|x^\perp|^2}$$
$$C_A = N \,, \qquad C_f = \frac{N^2 - 1}{2N} \,, \qquad w_A = -\frac{1}{12} \left(11 - 2\frac{N_f}{N} \right) \,, \qquad w_f = -\frac{3}{4} \,. \tag{2.14}$$

As an intermediate step we write the contributions of figure $1a$, $1b$, where some terms are still not well defined because of infrared divergencies.

$$\begin{aligned} &\int dz_1 dz_2 dz_{1'} dz_{2'} \left[\frac{\partial_1^2 \partial_{2'}^2 + \partial_{1'}^2 \partial_2^2}{(\partial_1 + \partial_{1'})^2} J_{111} + \frac{\partial_1 \partial_{2'} + \partial_{1'} \partial_2}{(\partial_1 + \partial_{1'})^2} \partial_1 \partial_2 J_0 \right] (A_1^* T^a A_{1'})(A_2^* T^a A_{2'}) + \\ &+ \left[\frac{\partial_{1'}^2 \partial_{2'}^2 + \partial_1^2 \partial_2^2}{(\partial_1 + \partial_{1'})^2} J_{111} + \frac{(\partial_{1'} \partial_{2'} + \partial_1 \partial_2)}{(\partial_1 + \partial_{1'})^2} \partial_1 \partial_2 \, J_0 \right] (A_1^* T^a A_{1'})(A_2 T^a A_{2'}^*) + \\ &+ \frac{(\partial_1 + \partial_{1'})^2}{\partial_1 \partial_2} J_{111} \, (A_1^* T^a A_{1'}^*) \, (A_2 T^a A_{2'}) - \\ &- \frac{(\partial_1 \partial_{1'} + \partial_2 \partial_{2'})}{(\partial_1 + \partial_2)^2} \, \partial_1 \partial_2 J_0 \, (A_1^* T^a A_2)(A_{1'} T^a A_{2'}^*) \end{aligned} \tag{2.15}$$

We abbreviate the dependence of the parton fields on the light ray position by subscripts, e.g. $A_1 = A(z_1)$. We sum the terms in the square brackets in order to cancel infrared divergencies by applying the relation of J integrals,

$$\begin{aligned} \partial_1 \partial_{2'} J_{111} + \partial_1 \partial_2 J_0 &= (\partial_1 + \partial_{1'})^2 \, \partial_{1'} \partial_{2'} \, J_{11'}^{(1)} \,, \\ J_{11'}^{(p)} &= -\int_0^1 d\alpha \frac{(1 - \alpha)^p}{\alpha} \delta(z_{11'} - \alpha z_{12}) \delta(z_{22'}) \,. \end{aligned} \tag{2.16}$$

The first square bracket transforms to $\partial_{1'} \partial_{2'} (J_{11'}^{(2)} + J_{22'}^{(2)})$, where $J_{22'}$ is obtained from $J_{11'}$ by the substitution $11' \leftrightarrow 22'$. The disconnected contribution, figure $1c$, amounts in adding

$$2 \left(\int_0^1 \frac{d\alpha}{\alpha} + w_A \right) \delta(z_{11'}) \delta(z_{22'}) \tag{2.17}$$

to both square brackets resulting in improving $J_{11'} + J_{22'}$ by the standard + prescription. We assume it to be included in the definition of $J_{11'}$ in the following.

This transformation brings the first bracket in the final form. The difference of the second from the first bracket,

$$-(\partial_{1'}-\partial_1)(\partial_{2'}-\partial_2)J_{111}+\partial_1\partial_2 J_0\,,$$

is then transformed by using the relations

$$\begin{aligned}
-(\partial_1\partial_{2'}+\partial_2\partial_{1'})J_{111} &= \partial_{1'}\partial_{2'}J_{112}+\partial_1\partial_2 J_0\,,\\
(\partial_1+\partial_{1'})^2 J_{111} &= -\partial_{1'}\partial_{2'}J_{221}-(\partial_1\partial_{2'}+\partial_2\partial_{1'})\frac{\partial_1\partial_2}{(\partial_1+\partial_2)^2}J_0\,.
\end{aligned}\tag{2.18}$$

We use the notations

$$\begin{aligned}
J_{n_1n_2n_3} &= \frac{\Gamma(n_1+n_2+n_3-1)}{\Gamma(n_1)\Gamma(n_2)\Gamma(n_3)}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\,\delta\left(\sum\alpha_i-1\right)\times\\
&\qquad\times\alpha_1^{n_1-1}\alpha_2^{n_2-1}\alpha_3^{n_3-1}\delta(z_{11'}-\alpha_1 z_{12})\delta(z_{22'}+\alpha_2 z_{12})\,,\\
J_0^{(p)} &= \left(\frac{\partial_1\partial_2}{(\partial_1+\partial_2)^2}\right)^p J_0\\
&= \int_0^1 d\alpha(\alpha(1-\alpha))^p\delta(z_{11'}-\alpha z_{12})\delta(z_{22'}+(1-\alpha)z_{12})\,.
\end{aligned}\tag{2.19}$$

The second relation in (2.18)) is also applied to the third term in (2.15). Then one observes the cancellation of the last term involving J_0 against other J_0 terms emerging from the transformations by (2.18). The cancellation is due to the crossing relation

$$(A_1T^aA_{1'})(A_2T^aA_{2'})-(A_1T^aA_{2'})(A_2T^aA_{1'})=(A_1T^aA_2)(A_{1'}T^aA_{2'})\tag{2.20}$$

implied by the Jacobi identity.

The final form of the effective gluon-gluon interaction vertices in the Bjorken asymptotics reads

$$\begin{aligned}
&\int 4\left[J_{11'}^{(2)}+J_{22'}^{(2)}+2w_A\delta(z_{11'})\delta(z_{22'})\right]\times\\
&\quad\times\left\{(\partial^{-1}A_1^*T^a\partial A_{1'})(\partial^{-1}A_2^*T^a\partial A_{2'})+(\partial^{-1}A_1^*T^a\partial A_{1'})(\partial^{-1}A_2T^a\partial A_{2'}^*)\right\}-\\
&\quad-4(J_{221}+2J_{112})(\partial^{-1}A_1^*T^a\partial A_{1'})(\partial^{-1}A_2T^a\partial A_{2'}^*)-\\
&\quad-4J_{221}(\partial^{-1}A_1^*T^a\partial A_{1'}^*)(\partial^{-1}A_2T^a\partial A_{2'})+[A\leftrightarrow A^*]\,.
\end{aligned}\tag{2.21}$$

The integrations run over the light ray positions z_i, $i=1,2,1',2'$.

For later comparison we write also the result for the effective fermion-fermion interaction (one flavour and one chirality only)

$$\begin{aligned}
&\int\left[J_{11'}^{(1)}+J_{22'}^{(1)}+2w_f\delta(z_{11'})\delta(z_{22'})\right]\times\\
&\quad\times\left\{(\partial^{-1}f_1^*t^af_{1'})(\partial^{-1}f_2^*t^af_{2'})+(\partial^{-1}f_1^*t^af_{1'})(\partial^{-1}f_2t^af_{2'}^*)\right\}-\\
&\quad-J_{111}(\partial^{-1}f_1^*t^af_{1'})(\partial^{-1}f_2t^af_{2'}^*)-\\
&\quad-2J_0^{(1)}(\partial^{-1}f_1^*t^af_2)(\partial^{-1}f_{1'}t^af_{2'}^*)+[f\leftrightarrow f^*]\,.
\end{aligned}\tag{2.22}$$

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2.2 Conformal symmetry

The conformal transformations of the light-ray position z and of the parton fields $A_p(z)$, $A_{p'}(z)$ are based on the Lie algebra $sl(2)$,

$$[S^0, S^{\pm}] = \pm S^{\pm}, \qquad [S^+, S^-] = 2S^0 . \tag{2.23}$$

The action on the light ray, infinitesimal Möbius transformations, are generated by

$$S^{(0),-} = -\partial , \qquad S^{(0)0} = z\partial , \qquad S^{(0)+} = +z^2\partial . \tag{2.24}$$

We need generic representations of conformal weight ℓ on functions of z generated by

$$S^{(\ell)-} = S^{(0)-} , \qquad S^{(\ell)0} = z^{-\ell} S^{(0)0} z^{\ell} , \qquad S^{(\ell)+} = z^{-2\ell} S^{(0)+} z^{2\ell} . \tag{2.25}$$

The representation space V_ℓ of single-parton wave functions $\psi(z)$ is spanned by monomials $\psi^{(m)} = z^m$, the constant function $\psi^{(0)} = 1$ represents the lowest weight state. (In this convention the negative integer and half-integer values of ℓ correspond to the finite-dimensional representations related to angular momentum and spin.)

Following [23], in V_ℓ a scalar product is introduced, defined on the basis by

$$\langle z^m, z^n \rangle_{(\ell)} = \delta_{n,m} \frac{\Gamma(m+1)\Gamma(2\ell)}{\Gamma(m+2\ell)} \tag{2.26}$$

It obeys the symmetry condition

$$\langle S^0\psi_1, \psi_2\rangle_{(\ell)} = \langle \psi_1, S^0\psi_2\rangle_{(\ell)} , \qquad \langle S^+\psi_1, \psi_2\rangle_{(\ell)} = -\langle \psi_1, S^-\psi_2\rangle_{(\ell)} . \tag{2.27}$$

The scalar product $\langle \psi_1, \psi_2\rangle_{(\ell)}$ can be calculated by the action of the differential operator $\hat{\psi}_1(\partial)$ on $\psi_2(z)$,

$$\langle \psi_1, \psi_2\rangle_{(\ell)} = \hat{\psi}_1(\partial)\, \psi_2(z)|_{z=0} , \tag{2.28}$$

where the symbol of this operator $\hat{\psi}(\zeta)$ is calculated from the function $\psi(z)$ as

$$\psi(z) \sum_m c_m z^m , \qquad \hat{\psi}(\zeta) = \sum_m c_m \frac{\Gamma(2\ell)}{\Gamma(m+2\ell)} \zeta^m . \tag{2.29}$$

The conformal symmetry properties of the multi-parton states $\psi(z_1, \ldots z_k)$ are represented on the tensor product $\otimes_1^k V_{\ell_i}$ and the generators (at tree level) are

$$S^a_{\{\ell_i\}} = \sum_{i=1}^k S^a_{\ell_i} . \tag{2.30}$$

Irreducible representations of weight $L = \sum_1^k \ell_i + n$, $n = 0, 1, \ldots$ are spanned by

$$\psi^{(m)}_{\{\ell_i\},n} = (S^+_{\{\ell_i\}})^m \, \psi^{(0)}_{\{\ell_i\},n} \tag{2.31}$$

with the lowest weight states obeying

$$S^0_{\{\ell_i\}}\, \psi^{(0)}_{\{\ell_i\},n} = \left(\sum_1^k \ell_i + n \right) \psi^{(0)}_{\{\ell_i\},n} ; \qquad S^-_{\{\ell_i\}}\, \psi^{(0)}_{\{\ell_i\},n} = 0 . \tag{2.32}$$

They are represented by a homogeneous polynomials of degree n of the differences $z_{ij} = z_i - z_j$. The symmetry isomorphism between the wave functions $\psi(z_1, \dots, z_k)$ and the differential operators $\hat{\psi}(\partial_1, \dots, \partial_k)$ induced by the scalar product allows to formulate the symmetric relation between the multiparton states and the composite operators of the parton fields. If the field operators A_i transform as conformal primaries of weight ℓ_i then the composite field operators

$$\hat{\mathcal{O}}(z) = \hat{\psi}(\partial_1, \dots, \partial_k) \prod_1^k A_i(z_i)|_{z_i \to z} \tag{2.33}$$

mix under one-loop renormalization only for those ψ belonging to one particular conformal symmetry representation weight L and one particular level m. With the lowest weight state, i.e. specifying to $\hat{\psi}^{(0)}_{\{\ell_i\},n}$, the corresponding operator on tree level transforms as the conformal primary of weight $L = \sum \ell_i + n$. The relevant QCD bare conformal primaries are the fermion field components $f, f^*, \tilde{f}, \tilde{f}^*$ with weight $\ell = 1$ and the gluon field strength components $\partial A, \partial A^*$ with weight $\ell = \frac{3}{2}$. In the case of two partons, $k = 2$, there is just one irreducible representation for each n and therefore $\psi^{(m)}_{\ell_1,\ell_2,n}$ are eigenstates of the evolution at one loop and the corresponding operators are multiplicatively renormalized.

Operators acting on two-parton states as $\hat{K}\psi(z_1, z_2) = \tilde{\psi}(z_1, z_2)$, $\hat{K} : V_{\ell_{1'}} \otimes V_{\ell_{2'}} \to V_{\ell_1} \otimes V_{\ell_2}$, are conformal symmetric if

$$(S^a_{\ell_1} + S^a_{\ell_2})\hat{K} = \hat{K}(S^a_{\ell'_1} + S^a_{\ell'_2}) \, . \tag{2.34}$$

Representing $\hat{K}$ in integral form

$$\hat{K}\psi(z_1, z_2) = \int_{C_{1'}, C_{2'}} dz_{1'} dz_{2'} K(z_1, z_2; z_{1'}, z_{2'}) \, \psi(z_{1'}, z_{2'}) \tag{2.35}$$

the symmetry condition on the kernel reads

$$\left(S^a_{\ell_1,1} + S^a_{\ell_2,2} - (S^a_{\ell'_1,1'})^T - (S^a_{\ell'_2,2'})^T \right) K(z_1, z_2; z_{1'}, z_{2'}) = 0 \, . \tag{2.36}$$

In the case of integration over closed contours the transposed generators are simply

$$(S^a_\ell)^T = -S^a_{\bar{\ell}} \, , \qquad \bar{\ell} = 1 - \ell \, , \tag{2.37}$$

and the kernel is a conformal symmetric 4-point function,

$$K_{(\ell_1,\ell_2,\bar{\ell}_{1'},\bar{\ell}_{2'})}(z_1, z_2; z_{1'}, z_{2'}) \sim \langle A_{\ell_1}(z_1) \, A_{\ell_2}(z_2) \, A_{\bar{\ell}_{1'}}(z_{1'}) \, A_{\bar{\ell}_{2'}}(z_{2'}) \rangle \, , \tag{2.38}$$

where A_ℓ stands for a conformal primary field operator of weight ℓ. A generic solution of the symmetry condition (2.36) reads

$$K^{(d,h)}_{(\ell_1,\ell_2,\bar{\ell}_{1'},\bar{\ell}_{2'})}(z_1, z_2; z_{1'}, z_{2'}) = z_{12}^{a_{12}} \, z_{1'2'}^{a_{1'2'}} \, z_{11'}^{a_{11'}} \, z_{22'}^{a_{22'}} \, z_{12'}^{a_{12'}} \, z_{21'}^{a_{21'}} \tag{2.39}$$

with the exponents depending on the weights and on two parameters d, h as

$$\begin{aligned} a_{12} &= d + \frac{1}{2}\sigma - \ell_1 - \ell_2 \,, & a_{1'2'} &= d + \frac{1}{2}\sigma - \bar{\ell}_{1'} - \bar{\ell}_{2'} \,, \\ a_{12'} &= h + \frac{1}{2}\sigma - \ell_1 - \bar{\ell}_{2'} \,, & a_{21'} &= h + \frac{1}{2}\sigma - \ell_2 - \bar{\ell}_{1'} \,, \\ a_{11'} &= -d - h - \ell_1 - \bar{\ell}_{1'} \,, & a_{22'} &= -d - h - \ell_2 - \bar{\ell}_{2'} \,, \\ \sigma &= \ell_1 + \ell_2 + \bar{\ell}_{1'} + \bar{\ell}_{2'} \,. \end{aligned} \tag{2.40}$$

Any symmetric 4-point function is a sum of such expressions with coefficients depending on the parameters d, h; it differs from (2.39) by a factor being a function of the anharmonic ratio $r_{st} = \frac{z_{11'} z_{22'}}{z_{12} z_{1'2'}}$. In (2.39) the powers with exponents d, h can be written as the powers of the anharmonic ratios $(r_{st})^{-d} (r_{ut} r_{st}^{-1})^h$ which are not independent,

$$r_{ut} = \frac{z_{12'} z_{21'}}{z_{12} z_{1'2'}} = r_{st} - 1 \,. \tag{2.41}$$

In view of the branch points appearing for non-integer a_{ij} we specify closed integration contours in (2.35) as the Pochhammer contours $C_{2'} = [2^+, 1'^+, 2^-, 1'^-]$, $C_{1'} = [2^+, 1^+, 2^-, 1^-]$, where e.g. 2^+ means to go around the point z_2 in positive sense.

Actually we should have the integration running over the light ray, i.e. the real axis. In the regular case, $a_{ij} > -1$, the contours can be contracted easily to the real axis on which z_1, z_2 are located. It is convenient to write the resulting kernel as

$$\begin{aligned} & z_{12}^{2-\sigma} \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(\sum \alpha_1 - 1\right) \alpha_1^{a_{11'}} \alpha_2^{a_{22'}} \alpha_3^{a_{1'2'}} (1-\alpha_1)^{a_{21'}} (1-\alpha_2)^{a_{22'}} \times \\ & \qquad \times \delta(z_{11'} - \alpha_1 z_{12}) \delta(z_{22'} + \alpha_2 z_{12}) \,. \end{aligned} \tag{2.42}$$

The contraction has to be done with more care in the singular cases. For example, if one exponent goes to -1 the result of the contraction is reproduced by the substitution in (2.42)

$$\begin{aligned} & \alpha^{-1+\varepsilon} \to \frac{1}{\varepsilon}\delta(\alpha) + \frac{1}{[\alpha]_+} \,, \\ & \int_0^1 d\alpha \frac{1}{[\alpha]_+} \phi(\alpha) = \int_0^1 d\alpha \frac{1}{\alpha} (\phi(\alpha) - \phi(0)) \,. \end{aligned} \tag{2.43}$$

Consider the case of equal weights, $\ell_1 = \ell_2 = \ell_{1'} = \ell_{2'} = \ell$. An example of singular contraction contributions arises for the parameters $d = \varepsilon \to 0$, $h = 0$. We obtain the kernel of the identity operator as the leading contribution in ε. The next term is proportional to $J_{11'}^{(2\ell-1)} + J_{22'}^{(2\ell-1)}$, with the notation defined in (2.16). For the parameters $h = 0$ and $-2\ell < d < 0$ (assuming $\ell > 0$) we have the regular case of contraction and obtain the kernel $J_{-d,-d,d+2\ell}$, with the notation given in (2.19)). Another singular case of contraction arises at $d = -2\ell + \varepsilon$, the leading term resulting in $J_0^{(2\ell-1)}$, with the notation given in (2.19)). The latter examples cover the kernels encountered in the QCD result above, for the particular parameter values

$$h = 0 \,, \qquad d = -\varepsilon \,, \qquad d = -1 \,, \qquad d = -2(+\varepsilon)$$

and with the conformal weights $\ell = 3/2$ for the gluon-gluon vertices and $\ell = 1$ for the quark-quark vertices.

Also the kernels appearing in the mixed gluon-quark and annihilation-type vertices can be identified, with the appropriate choice of the weights, as particular cases of conformal 4-point functions [16].

Generating the evolution in the Bjorken limit the effective vertices (2.21) can be considered as second-quantized parton field operators by reading $A_{p'}$ as annihilation and A_p as creation operators with the contraction rules

$$\langle A_1^*(z_1)A_{1'}(z_{1'})\rangle = \langle A_1(z_1)A_{1'}^*(z_{1'})\rangle = \frac{1}{2}\delta(z_{11'})\,.$$

Two parton states onto which these operators act are defined by the wave function, e.g. for two gluons of helicities $+1$ and -1

$$|n,m;A^*,A\rangle = \int_{-\infty}^{+\infty} dz_1 dz_2 \psi_n^{(m)}(z_1,z_2)\partial^{-1}A_1^*(z_1)\partial^{-1}A_2(z_2)|0\rangle\,. \tag{2.44}$$

The label A stands for parton type (gluon) and helicity (-1). For representing the action of the vertices on the composite operators (2.33) we read the involved parton fields as annihilation operators $(A_{p'})$. It is clear that the eigenstates have the form $|n,m;A^*,A\rangle \pm |n.m;A,A^*\rangle$, $|n,m;A^*,A^*\rangle$, $|n.m;A,A\rangle$. They contribute, respectively, to the t channel exchange in cases of unpolarized, polarized (helicity asymmetry) and transversity (generalized) parton distributions.

3. Parton interaction in SYM

The kinetic terms of the super Yang-Mills theory are obtained from the ones of QCD (2.5) by changing the fermion representation to the adjoint ($f^\alpha \to f^a, t^\alpha \to T^a$) and by restricting to one flavour and one chiral component (i.e. omitiing the $\tilde{f}$ term). We introduce superfields,

$$\mathcal{A}^a(x,\theta) = A^a(x) + c\theta f^a(x)\,, \qquad \mathcal{A}^{*a} = A^{*a} + c\theta f^{*a}\,, \qquad c = \frac{1}{\sqrt{2i}} \tag{3.1}$$

in order to write the kinetic terms as

$$\begin{aligned} S_{\rm kin}^{\rm SYM} &= -2\int d^4x d\theta \mathcal{A}^{*a}\partial^{-1}D(\partial_+\partial_- - \partial^\perp\partial^{*\perp})\mathcal{A}^a \\ &\qquad D = \partial_\theta + \theta\partial\,, \qquad D^2 = \partial\,. \end{aligned} \tag{3.2}$$

This action is invariant with respect to super translations generated by $S^{-\frac{1}{2}} = -\partial_\theta + \theta\partial$. The interaction terms can be reconstructed by super gauge symmetry, i.e. by extending the transverse derivatives by

$$\partial^\perp\mathcal{A}^a \to (\mathcal{D}^\perp\mathcal{A})^a = \partial^\perp\mathcal{A}^a + \frac{ig}{2}(\mathcal{A}^*T^a A)\,, \tag{3.3}$$

and replacing the integrand in the kinetic term as

$$\mathcal{A}^{*a}D\partial_-\ \mathcal{A}^a - (\mathcal{D}^{*\perp}D\mathcal{A}^*)^a\partial^{-2}D(\mathcal{D}^\perp D\mathcal{A})^a\,. \tag{3.4}$$

The result is

$$
\begin{aligned}
&S^{\rm SYM}_{\rm kin}+S^{\rm SYM}_3+S^{\rm SYM}_4\,,\\
&\qquad S^{\rm SYM}_3=\frac{g}{2}\int d^4x d\theta(\partial_1\hat V^*_{123}D_1\mathcal{A}^a_1(\mathcal{A}^*_2T^a\mathcal{A}^*_3|_{(x_i,\theta_i)=(x,\theta)}+\text{c.c.}),\\
&\qquad S^{\rm SYM}_4=\frac{g^2}{4}\int d^4x d\theta(\mathcal{A}^*_1T^cD\mathcal{A}_{1'})\partial^{-2}D(D\mathcal{A}^*_2T^c\mathcal{A}_{2'})|_{(x_i,\theta_i)=(x,\theta)}\,.
\end{aligned}
\tag{3.5}
$$

We have abbreviated the dependence of the superfields on spacetime position x_i and θ_i by subscript $i=1,2,1',2'$.

The close similarity to the gluodynamic case allows to follow the previous calculation with minor modifications. The calculation in terms of superfields preserves the symmetry with respect to the super translation $S^{-\frac{1}{2}}=-\partial_\theta+z\partial$ in all steps. We use the super propagators

$$
\begin{aligned}
\langle\mathcal{A}^*_1\mathcal{A}_2\rangle&\sim(x_{+12}\tilde z_{12}-|x^\perp_{12}|^2)^{-2}\,,\\
\langle\mathcal{A}^*_1\partial^{-1}D\mathcal{A}_2\rangle&\sim\theta_{12}(x_{+12}z_{12}-|x^\perp_{12}|^2)^{-2}\,.\\
&\qquad x_{12}=x_1-x_2\,,\quad\tilde z_{12}=z_1-z_2-\theta_1\theta_2\,,\quad\theta_{12}=\theta_1-\theta_2\,.
\end{aligned}
\tag{3.6}
$$

The resulting effective vertices can be represented in a form close to the one of $S^{\rm SYM}_4$. The result for figure $1a$, $1b$ is

$$
\begin{aligned}
&\int dz_1dz_2dz_{1'}dz_{2'}d\theta_1d\theta_2d\theta_{1'}d\theta_{2'}\times\\
&\quad\times\left[\frac{\partial_1D_1\partial_{2'}D_{2'}+\partial_{1'}D_{1'}\partial_2D_2}{(\partial_1+\partial_{1'})^2}\theta_{11'}\theta_{22'}\tilde J_{111}+\frac{D_1D_{2'}+D_{1'}D_2}{(\partial_1+\partial_{1'})^2}\partial_1\partial_2\theta_{11'}\theta_{22'}\tilde J_0\right]\times\\
&\quad\times(\mathcal{A}^*_1T^a\mathcal{A}_{1'})(\mathcal{A}^*_2T^a\mathcal{A}_{2'})+\\
&\quad+\left[\frac{\partial_{1'}D_{1'}\partial_{2'}D_{2'}+\partial_1D_1\partial_2D_2}{(\partial_1+\partial_{1'})^2}\theta_{11'}\theta_{22'}\tilde J_{111}+\frac{D_{1'}D_{2'}+D_1D_2}{(\partial_1+\partial_{1'})^2}\partial_1\partial_2\;\theta_{11'}\theta_{22'}\tilde J_0\right]\times\\
&\quad\times(\mathcal{A}^*_1T^a\mathcal{A}_{1'})(\mathcal{A}_2T^a\mathcal{A}^*_{2'})-\\
&\quad-(D_1+D_{1'})(D_2+D_{2'})\theta_{11'}\theta_{22'}\tilde J_{111}(\mathcal{A}^*_1T^a\mathcal{A}^*_{1'})(\mathcal{A}_2T^a\mathcal{A}_{2'})-\\
&\quad-\frac{(D_1D_{1'}+D_2D_{2'})}{(\partial_1+\partial_2)^2}\,\partial_1\partial_2\theta_{12}\theta_{1'2'}\tilde J_0(\mathcal{A}^*_1T^a\mathcal{A}_2)(\mathcal{A}_{1'}T^a\mathcal{A}^*_{2'})\,.
\end{aligned}
\tag{3.7}
$$

Here the $\mathcal{A}_i$ denote the parton super field with the subscript abbreviating the argument (z_i,θ_i). The kernels with tilde $\tilde J$ are obtained from the corresponding kernels J defined in section 2 by replacing the distances $z_{11'},z_{22'},z_{1'2'}$ by their supersymmetric extensions $\tilde z_{11'}=z_{11'}-\theta_1\theta_{1'},\tilde z_{22'},\tilde z_{1'2'}$. The integrand of the SYM results differs from the one of the gluodynamic result by merely the latter replacement, factors $\theta_{11'}\theta_{22'}$ or $\theta_{12}\theta_{1'2'}$ and the replacement of some derivatives ∂ by D.

4. Superconformal symmetry

The superconformal transformations of the extended light ray (z,θ) and of the superfields residing on it relies on the algebra $osp(2|1),S^a,a=0,\pm\frac{1}{2},\pm1$,

$$
\begin{aligned}
&[S^0,S^a]=aS^a,\qquad[S^{+1},S^{-1}]=2S^0,\qquad[S^{-\frac{1}{2}},S^{+1}]=S^{+\frac{1}{2}},\quad[S^{+\frac{1}{2}},S^{-1}]=S^{-\frac{1}{2}},\\
&[S^{-\frac{1}{2}},S^{-\frac{1}{2}}]_+=2S^{-1},\quad[S^{+\frac{1}{2}},S^{+\frac{1}{2}}]_+=-2S^{+1},\quad[S^{+\frac{1}{2}},S^{-\frac{1}{2}}]_+=-2S^0.
\end{aligned}
\tag{4.1}
$$

From the last 3 relations we see that the algebra is generated by $S^{\pm\frac{1}{2}}$. Super Möbius transformations of the extended light ray are generated by the particular representation $S^a_0, \ell = 0$, where

$$S_0^{-\frac{1}{2}} = -\partial_\theta + \theta\partial\,, \qquad S_0^{+\frac{1}{2}} = zS_0^{-\frac{1}{2}}\,, \tag{4.2}$$

and the remaining generators following from the last 3 commutation relations (4.1). Representations on the fields are labeled by the weight ℓ and are generated by S^a_ℓ,

$$S_\ell^{-\frac{1}{2}} = S_0^{-\frac{1}{2}}\,, \qquad S_\ell^{+\frac{1}{2}} = z^{-2\ell} S_0^{+\frac{1}{2}} z^{2\ell}\,. \tag{4.3}$$

The corresponding representation space of 1-parton states, $\psi(z,\theta) \in V_\ell$, is spanned by $z^m, \theta z^m, m = 0, 1, \ldots$. The constant function represents the lowest weight state. A scalar product on V_ℓ with the symmetry properties

$$\begin{aligned} \langle S^0\psi_1, \psi_2\rangle_{(\ell)} &= \langle \psi_1, S^0\psi_2\rangle_{(\ell)}\,, \\ \langle S^a\psi_1, \psi_2\rangle_{(\ell)} &= -\langle \psi_1, S^a\psi_2\rangle_{(\ell)}\,, \qquad a = \pm\frac{1}{2}, \pm 1\,. \end{aligned} \tag{4.4}$$

is defined on the basis by

$$\begin{aligned} \langle z^m, z^n\rangle_{(\ell)} &= \delta_{n,m}\frac{\Gamma(m+1)\Gamma(2\ell)}{\Gamma(m+2\ell)} \\ \langle \theta z^m, \theta z^n\rangle_{(\ell)} &= \delta_{n,m}\frac{\Gamma(m+1)\Gamma(2\ell)}{\Gamma(m+1+2\ell)} \\ \langle \theta z^m, z^n\rangle_{(\ell)} &= \langle z^m, \theta z^n\rangle_{(\ell)} = 0\,. \end{aligned} \tag{4.5}$$

It can be calculated by differentiations,

$$\langle \psi_1, \psi_2\rangle_{(\ell)} = \hat{\psi}_1(\partial, D)\psi_2(z,\theta)|_{z=0,\theta=0}\,, \tag{4.6}$$

where the symbol of the differential operator $\hat{\psi}(y,\vartheta)$ is calculated from the expansion of the function $\psi(z,\theta)$,

$$\begin{aligned} \psi(z,\theta) &= \sum_m (a_m + \theta b_m) z^m\,, \\ \hat{\psi}(y,\vartheta) &= \Gamma(2\ell)\sum_m \left(a_m + \vartheta\frac{b_m}{2\ell+m}\right)\frac{1}{\Gamma(m+2\ell)} y^m\,. \end{aligned} \tag{4.7}$$

The action of S^a_ℓ on $\psi(z,\theta)$ is isomorphic to the action of $\hat{S}^a_\ell$ on $\hat{\psi}(y,\vartheta)$, where

$$\hat{S}_\ell^{-\frac{1}{2}} = -y\partial_y\partial_\vartheta + \vartheta\partial_y - 2\ell\partial_\vartheta\,, \qquad \hat{S}_\ell^{+\frac{1}{2}} = \vartheta - y\partial_\vartheta\,, \tag{4.8}$$

and $\hat{S}^a_\ell$ obey (4.1).

Multi (k) parton states are represented by functions of (z_i, θ_i), $i = 1, \ldots, k$, $\psi(z_1, \theta_1, \ldots, z_k, \theta_k) \in \otimes_1^k V_{\ell_i}$ and the transformations are generated by the sum $\sum_{i=1}^k S^a_{\ell_i} = S^a_{\{\ell_i\}}$.

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The lowest weight states of the representations into which the tensor product decomposes obey

$$S^{-\frac{1}{2}}_{\{\ell_i\}}\psi^{(0)}_{\{\ell_i\},n}(z_1,\theta_1,\dots,z_k,\theta_k)=0\,,\quad S^{0}_{\{\ell_i\}}\psi^{(0)}_{\{\ell_i\},n}=\left(\sum\ell_i+n\right)\psi^{(0)}_{\{\ell_i\},n}\,,\qquad n=0,\frac{1}{2},1,\dots \tag{4.9}$$

and, therefore, are homogeneous functions of z_i,θ_i of degree n of the super-translation invariant combinations

$$\tilde{z}_{ij}=z_i-z_j-\theta_i\theta_j\,,\qquad \theta_{ij}=\theta_i-\theta_j\,. \tag{4.10}$$

In the particular case of $k=2$ we have with the notations $\bar{n}$ = integer part of n and $\nu=2(n-\bar{n})$

$$\psi^{(0)}_{\ell_1,\ell_2,\bar{n},0}=\tilde{z}^{\bar{n}}_{12}\,,\qquad \psi^{(0)}_{\ell_1,\ell_2,\bar{n},1}=\theta_{12}z^{\bar{n}}_{12}\,. \tag{4.11}$$

and the irreducible representations are spanned by

$$\psi^{(m)}_{\ell_1,\ell_2,\bar{n},\nu}=(S^{+\frac{1}{2}}_{\ell_1}+S^{+\frac{1}{2}}_{\ell_2})^m\psi^{(0)}_{\ell_1,\ell_2,\bar{n},\nu}\,,\qquad m=0,\frac{1}{2},1,\dots \tag{4.12}$$

in particular, for the next-to-lowest level $m=1/2$ we have for $\ell_1=\ell_2=\ell$,

$$\psi^{(\frac{1}{2})}_{\ell,\ell,\bar{n},0}=(\bar{n}+2\ell)(\theta_1+\theta_2)z^{\bar{n}}_{12}\,,\qquad \psi^{(\frac{1}{2})}_{\ell,\ell,\bar{n},1}=-z^{\bar{n}+1}_{12}-(\bar{n}+4\ell)\theta_1\theta_2z^{\bar{n}}_{12}\,. \tag{4.13}$$

The relation between parton states ψ and composite operators mentioned above generalizes to the supersymmetric case,

$$\hat{\mathcal{O}}(z,\theta)=\hat{\psi}(\partial_1,D_1,\dots,\partial_k,D_k)\prod_1^k\mathcal{A}_i(z_i,\theta_i)|_{z_i,\theta_i)\to(z,\theta)}\,. \tag{4.14}$$

Also the notion of symmetric operators $\hat{K}$ carries over from the case of $sl(2)$ to the case of $osp(2|1)$. If $\hat{K}$ is given in integral form

$$\hat{K}\psi(z_1,\theta_1,z_2,\theta_2)=\int_{C_{1'},C_{2'}}dz_{1'}dz_{2'}d\theta_{1'}d\theta_{2'}K(z_1,\theta_1,z_2,\theta_2;z_{1'},\theta_{1'},z_{2'},\theta_{2'})\psi(z_{1'},\theta_{1'}z_{2'},\theta_{2'}) \tag{4.15}$$

then the symmetry conditions on the kernel is analogous to (2.36), where here the particular conditions with $a=\pm1/2$ are sufficient. For closed contours (in $z_{1'},z_{2'}$) the transposition acts on the generators as

$$(S^a_\ell)^T=-S^a_{\bar{\ell}}\,,\qquad \bar{\ell}=\frac{1}{2}-\ell\,, \tag{4.16}$$

where the relation between the weights $\ell,\bar{\ell}$ differs from the non-supersymmetric case (2.37). The particular condition on the kernel with $a=-1/2$ implies that the kernel K depends on the supertranslation invariant combinations $\tilde{z}_{ij},\theta_{ij},(i,j=1,2,1',2')$ only. The remaining independent condition with $a=+1/2$ is solved by an expression $\tilde{K}^{(d,h)}_{\ell_1,\ell_2,\bar{\ell}_{1'},\bar{\ell}_{2'}}$ of the form (2.39) depending on two parameters d,h, and on the weights $\ell_1,\ell_2,\ell_{1'},\ell_{2'}$ in the same way (2.40) but with z_{ij} replaced by $\tilde{z}_{ij}$. The powers with the exponents d, h can be written as powers of the anharmonic ratios likewise,

$$\tilde{r}_{st} = \frac{\tilde{z}_{11'}\tilde{z}_{22'}}{\tilde{z}_{12}\tilde{z}_{1'2'}}, \qquad \tilde{r}_{ut} = \frac{\tilde{z}_{12'}\tilde{z}_{21'}}{\tilde{z}_{12}\tilde{z}_{1'2'}}. \tag{4.17}$$

Two symmetric 4-point functions of the same set of weights differ by a factor being a function of invariants built out of the coordinates of the 4 points in complete analogy to the $sl(2)$ case. The difference is that now out of the coordinates of 4 points (z_i, θ_i) one can construct two independent invariants. In particular the two anharmonic ratios $\tilde{r}_{st}, \tilde{r}_{ut}$ are not dependent (unlike (2.41)), but obey

$$\begin{aligned} \tilde{r}_{st} - \tilde{r}_{ut} - 1 &= r_{\theta t}, \\ r_{\theta t} &= \frac{1}{\tilde{z}_{12}\tilde{z}_{1'2'}} \left(\tilde{z}_{22'}\theta_{12'}\theta_{11'} + \tilde{z}_{11'}\theta_{21'}\theta_{22'} + \tilde{z}_{1'2'}\theta_{11'}\theta_{22'}\right). \end{aligned} \tag{4.18}$$

We conclude that the from (2.39) (with z_{ij} replaced by $\tilde{z}_{ij}$) represents a generic form of $(osp(2|1))$ superconformal 4-point functions. Sums with varying values of d, h represent all of them. Alternatively, one can consider

$$(A_d + B_d r_{\theta t})\, \tilde{K}^{(d,0)}_{\ell,\ell_2,\bar{\ell}_{1'},\bar{\ell}_{2'}} \tag{4.19}$$

as the generic form and represent arbitrary symmetric 4-point functions as sums of them with varying parameter d.

For our applications we may restrict ourselves to the case $\ell_1, \ell_2, \bar{\ell}_{1'}, \bar{\ell}_{2'} = \ell = 1$. The basic two-parton wave functions $\psi^{(m)}_{\ell,\ell,\bar{n},\nu}$ are eigenfunctions

$$\int_{C_{1'},C_{2'}} dz_{1'}dz_{2'}d\theta_{1'}d\theta_{2'}\,(A_d + B_d r_{\theta t})\, \tilde{K}^{(d,0)}_{(\ell)} \psi^{(m)}_{\ell,\ell,\bar{n},\nu} = (A_d \lambda^{(d,0)}_{\ell,\bar{n},\nu} + B_d \lambda^{(d,\theta)}_{\ell,\bar{n},\nu})\psi^{(m)}_{\ell,\ell,\bar{n},\nu}. \tag{4.20}$$

The eigenvalues do not depend on the level m.

$$\begin{aligned} \lambda^{(d,0)}_{\ell,\bar{n},\nu} &= (-1)^\nu \frac{\Gamma(d + \bar{n} + 2\ell + \frac{1}{2})}{\Gamma(-d + \bar{n} + 2\ell + \frac{1}{2})} \Gamma\left(-d + \frac{1}{2}\right)^2, \\ \lambda^{(d,\theta)}_{\ell,\bar{n},\nu} &= -\frac{\Gamma(d + \bar{n} + 2\ell - \frac{1}{2} + \nu)}{\Gamma(-d + \bar{n} + 2\ell + \frac{1}{2} + \nu)} \Gamma\left(-d + \frac{1}{2}\right)^2. \end{aligned} \tag{4.21}$$

The dependence on $\bar{n}, \nu$ can be calculated before the contours have been contracted to the real axis by using the contour integral representation of the Beta function. In the regular case the contraction results in the kernels

$$\begin{aligned} \tilde{K}^{(d,0)}_{(\ell)} &= \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(\sum \alpha_i - 1\right) \alpha_1^{-\frac{1}{2}-d} \alpha_2^{-\frac{1}{2}-d} \alpha_3^{d+2\ell-\frac{1}{2}} \times \\ &\quad \times \delta(\tilde{z}_{11'} - \alpha_1[\tilde{z}_{11'} + \tilde{z}_{1'2'} + \tilde{z}_{2'2}])\delta(\tilde{z}_{22'} + \alpha_2[\tilde{z}_{11'} + \tilde{z}_{1'2'} + \tilde{z}_{2'2}]) \times \\ &\quad \times \tilde{z}_{12}^{d-2\ell+\frac{1}{2}}[\tilde{z}_{12} + \theta_{12'}\theta_{21'}]^{-d+2\ell+\frac{1}{2}} \\ r_{\theta t}\tilde{K}^{(d,0)}_{(\ell)} &= \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(\sum \alpha_i - 1\right) \alpha_1^{-\frac{1}{2}-d} \alpha_2^{-\frac{1}{2}-d} \alpha_3^{d+2\ell-\frac{3}{2}} \times \\ &\quad \times \delta(\tilde{z}_{11'} - \alpha_1[\tilde{z}_{11'} + \tilde{z}_{1'2'} + \tilde{z}_{2'2}])\delta(\tilde{z}_{22'} + \alpha_2[\tilde{z}_{11'} + \tilde{z}_{1'2'} + \tilde{z}_{2'2}]) \times \\ &\quad \times [\alpha_1\theta_{21'}\theta_{22'} + \alpha_2\theta_{11'}\theta_{12'} + \alpha_3\theta_{11'}\theta_{22'}]. \end{aligned} \tag{4.22}$$

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For real $\ell > 0$ the regularity condition is $\frac{1}{2} > d > \frac{1}{2} - 2\ell$. The kernel of the identity operator $\delta(z_{11'})\theta_{11'}\delta(z_{22'})\theta_{22'}$ is in $r_{\theta t}\tilde{K}_\ell^{(d,0)}$ as the leading term in ε for $d \to +\frac{1}{2} - \varepsilon$. The next term $\sim \varepsilon^{-1}$ is proportional to

$$\begin{aligned} J_T^{(\ell)} &= J_{T1}^{(\ell)} - J_{T2}^{(\ell)} \\ J_{T1}^{(\ell)} &= -\int_0^1 d\alpha \left[\frac{(1-\alpha)^{2\ell}}{[\alpha]_+} \theta_{11'}\theta_{22'} + (1-\alpha)^{2\ell-1}\theta_{21'}\theta_{22'} \right] \times \\ &\quad \times \delta(\tilde{z}_{11'} - \alpha[\tilde{z}_{11'} + \tilde{z}_{1'2'} + \tilde{z}_{2'2}])\delta(\tilde{z}_{22'}) \end{aligned} \tag{4.23}$$

J_{T2} is obtained from J_{T1} by the interchange of subscripts $11' \leftrightarrow 22'$. For $d \to \frac{1}{2} - 2\ell$ we find both singular and regular contraction contributions in the decomposion with respect to the odd (θ) coordinates; e.g.

$$J_V^{(\ell)} = \lim_{\varepsilon \to 0} \; 2\tilde{K}_{(\ell)}^{(\frac{1}{2}-2\ell+\varepsilon,0)} \tag{4.24}$$

involves in the term proportional to $\theta_{1'}\theta_{2'}$ a contribution from singular contraction (by applying (2.43)) finite in the limit, whereas the terms in the remaining θ components are regular contraction contributions.

5. Symmetric form of the SYM parton interaction

In our application we have $\ell = 1$, the corresponding labels referring to the conformal weight will be omitted in the following.

The square bracket in first term in (3.7) can be written as J_T (4.23). Indeed, after applying (2.16) and adding the disconnected contributions allowing to write this expression in terms of $\tilde{J}_{11'}^{(1)}$ and $\tilde{J}_{22'}^{(1)}$ we have

$$\partial_{1'} D_1 D_{2'} \theta_{11'}\theta_{22'} \tilde{J}_{11'}^{(1)} = -D_{1'} D_{2'} \{ D_1 D_{1'} \theta_{11'}\theta_{22'} \tilde{J}_{11'}^{(1)} \} = D_{1'} D_{2'} \partial_{1'} J_{T1} \tag{5.1}$$

by comparing the results of the differentiations $D_1 D_{1'}$ in the braces with $\partial_{1'} J_{T1}$. We can check also the coincidence of corresponding coefficients in the expansion with respect to the odd coordinates. The one at $\theta_1\theta_2$ is proportional to $J_{11'}^{(2)} + J_{22'}^{(2)}$ and the one at $\theta_{1'}\theta_{2'}$ is proportional to $J_{11'}^{(1)} + J_{22'}^{(1)}$.

By applying the same transformation to the second square bracket in (3.7) it becomes

$$\begin{aligned} &D_{1'} D_{2'} J_T + \frac{(\partial_1 D_1 - \partial_{1'} D_{1'})(\partial_2 D_2 - \partial_{2'} D_{2'})}{(\partial_1 + \partial_{1'})^2} \theta_{11'}\theta_{22'} \tilde{J}_{111} - \\ &-(D_1 + D_{1'})(D_2 + D_{2'}) \frac{\partial_1 \partial_2}{(\partial_1 + \partial_2)^2} \theta_{11'}\theta_{22'} \tilde{J}_0 \,. \end{aligned} \tag{5.2}$$

By the crossing relation (2.20) we write the last term in (3.7) with $(\mathcal{A}_1^* T^a \mathcal{A}_2)(\mathcal{A}_{1'} T^a \mathcal{A}_{2'}^*)$ as two terms to be added to the one with $(\mathcal{A}_1^* T^a \mathcal{A}_{1'})$ $(\mathcal{A}_2 T^a \mathcal{A}_{2'}^*)$ and $(\mathcal{A}_1^* T^a \mathcal{A}_{1'}^*)$ $(\mathcal{A}_2 T^a \mathcal{A}_{2'})$.

As the result (3.7) reads

$$
\begin{aligned}
&J_T\{(\mathcal{A}_1^* T^a D\mathcal{A}_{1'})(\mathcal{A}_2^* T^a D\mathcal{A}_{2'}) + (\mathcal{A}_1^* T^a D\mathcal{A}_{1'})(\mathcal{A}_2 T^a D\mathcal{A}_{2'}^*)\} + \\
&+\bigg\{ \frac{(\partial_1 D_1 - \partial_{1'} D_{1'})(\partial_2 D_2 - \partial_{2'} D_{2'})}{(\partial_1 + \partial_{1'})^2}\theta_{11'}\theta_{22'}\tilde{J}_{111} - \\
&\quad - (D_1 + D_{1'})(D_2 + D_{2'})\frac{\partial_1\partial_2}{(\partial_1+\partial_2)^2}\theta_{11'}\theta_{22'}\tilde{J}_0 + \\
&\quad + (D_1 D_{1'} + D_2 D_{2'})\frac{\partial_1\partial_2}{(\partial_1+\partial_2)^2}\theta_{12}\theta_{1'2'}\tilde{J}_0\bigg\}(\mathcal{A}_1^* T^a D\mathcal{A}_{1'})(\mathcal{A}_2 T^a D\mathcal{A}_{2'}^*) - \\
&-\bigg\{(D_1 + D_{1'})(D_2 + D_{2'})\theta_{11'}\theta_{22'}\tilde{J}_{111} + \\
&\quad + (D_1 D_{2'} + D_2 D_{1'})\frac{\partial_1\partial_2}{(\partial_1+\partial_2)^2}\theta_{12}\theta_{1'2'}\tilde{J}_0\bigg\}(\mathcal{A}_1^* T^a D\mathcal{A}_{1'}^*)(\mathcal{A}_2 T^a D\mathcal{A}_{2'})\,. \qquad (5.3)
\end{aligned}
$$

Now we look for expressions of the remaining terms in (5.3), besides of the ones going with J_T, in terms of conformal symmetric kernels. We try in particular the kernels with weight $\ell = 1$ and $d = -1/2$ defining

$$
J_{V+A} = (1 + 2r_{\theta t})\tilde{K}^{(-\frac{1}{2},0)} \qquad (5.4)
$$

and the kernel at the lower limit of the regularity region ($d \to -3/2$) J_V (4.24) because in their odd coordinate decomposition the kernels of the QCD result (2.21), (2.22) appear.

Indeed we observe that the following relations hold

$$
\begin{aligned}
&(\partial_1 D_1 - \partial_{1'} D_{1'})(\partial_2 D_2 - \partial_{2'} D_{2'})\theta_{11'}\theta_{22'}\tilde{J}_{111} - \\
&\quad -(D_1 + D_{1'})(D_2 + D_{2'})\partial_1\partial_2\theta_{11'}\theta_{22'}\tilde{J}_0 + \\
&+(D_1 D_{1'} + D_2 D_{2'})(\partial_1 + \partial_{1'})^2\frac{\partial_1\partial_2}{(\partial_1+\partial_2)^2}\theta_{12}\theta_{1'2'}\tilde{J}_0 = -(\partial_1+\partial_{1'})^2 D_{1'} D_{2'}(J_V + 2J_{V+A})\,, \\
&(D_1 + D_{1'})(D_2 + D_{2'})\theta_{11'}\theta_{22'}\tilde{J}_{111} + \\
&\quad + (D_1 D_{2'} + D_2 D_{1'})\frac{\partial_1\partial_2}{(\partial_1+\partial_2)^2}\theta_{12}\theta_{1'2'}\tilde{J}_0 = D_{1'} D_{2'} J_V\,. \qquad (5.5)
\end{aligned}
$$

As the result we obtain the manifest superconformal expression of the SYM effective vertices in the Bjorken limit

$$
\begin{aligned}
&\int [J_T + 2w_f\theta_{11'}\theta_{22'}\delta(z_{11'})\delta(z_{22'})] \\
&\quad \{(\partial^{-1}\mathcal{A}_1^* T^a D\mathcal{A}_{1'})(\partial^{-1}\mathcal{A}_2^* T^a D\mathcal{A}_{2'}) + (\partial^{-1}\mathcal{A}_1^* T^a D\mathcal{A}_{1'})(\partial^{-1}\mathcal{A}_2 T^a D\mathcal{A}_{2'}^*)\} - \\
&\quad - [J_V + 2J_{V+A}](\partial^{-1}\mathcal{A}_1^* T^a D\mathcal{A}_{1'})(\partial^{-1}\mathcal{A}_2 T^a D\mathcal{A}_{2'}^*) - \\
&\quad - J_V(\partial^{-1}\mathcal{A}_1^* T^a D\mathcal{A}_{1'}^*)(\partial^{-1}\mathcal{A}_2 T^a D\mathcal{A}_{2'}) + (\mathcal{A} \leftrightarrow \mathcal{A}^*)\,. \qquad (5.6)
\end{aligned}
$$

The integration is over the super coordinates (z_i, θ_i) of the points $i = 1, 2, 1', 2'$. The dependence of the super fields $\mathcal{A}_i$ on the points is abbreviated by the corresponding subscript. The notation of the brackets with the generators T^a is as in (2.6). The superconformal kernels J_T, J_V, J_{V+A} depend on the coordinates of the 4 points and have been defined in (4.23), (4.24), (5.4).

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The corresponding result of [20] concerns the first term in (5.6) proportional to J_T, the remaining terms cannot be obtained as a reduction of the one of $\mathcal{N}=4$ SYM.

Supermultiplet parton states are defined in straightforward analogy to the two-gluon parton states, e.g.

$$|\bar{n},\nu,m;+,-\rangle = \int dz_1 dz_2 d\theta_1 d\theta_2 \psi^{(m)}_{\bar{n},\nu}(z_1,\theta_1,z_2,\theta_2)\partial^{-1}\mathcal{A}_1^*\partial^{-1}\mathcal{A}_2|0\rangle \tag{5.7}$$

Now the superfields at the primed position variables $(1',2')$ are considered as annihilation and the other ones $(1,2)$ as creation operators with the contraction rule $\langle D\mathcal{A}_{1'}\partial^{-1}\mathcal{A}_1^*\rangle \sim \delta(z_{11'})\theta_{11'}$.

Here it is enough to label the helicity states (by $\pm$) besides of the quantum numbers of the superconformal representation $\bar{n},\nu$; $(\bar{n}=0,1,\ldots,\nu=0,1)$ and the representation level $m=0,\frac{1}{2},1,\ldots$. Eigenstates of the effective vertex operator (5.6) are $|\bar{n},\nu,m;+,-\rangle \pm |\bar{n},\nu,m;+,-\rangle, |\bar{n},\nu,m;+,+\rangle|\bar{n},\nu,m;-,-\rangle$. The eigenvalues can be obtained by (4.21). The eigenvalues of $|\bar{n},\nu,m;+,+\rangle$ are proportional to

$$\lambda^T_{\bar{n},\nu} = 4[\psi(\bar{n}+2+\nu)-\psi(1)+w_f]\,. \tag{5.8}$$

The antisymmetric helicity states $|\bar{n},\nu,m;+,-\rangle - |\bar{n},\nu,m;-,+\rangle$ have the eigenvalues proportional to

$$\lambda^A_{\bar{n},\nu} = \lambda^T_{\bar{n},\nu} + 2\left(\frac{(-1)^\nu}{\bar{n}+2} - \frac{2}{(\bar{n}+2+\nu)(\bar{n}+1+\nu)}\right). \tag{5.9}$$

The additional term emerges from the term with J_{V+A} in (5.6). The symmetric helicity states $|\bar{n},\nu,m;+,-\rangle + |\bar{n},m;\nu,-,+\rangle$ have the eigenvalues proportional to

$$\lambda^V_{\bar{n},\nu} = \lambda^A_{\bar{n},\nu} + \frac{4(-1)^\nu}{(\bar{n}+3)(\bar{n}+2)(\bar{n}+1)}\,. \tag{5.10}$$

The additional term emerges from the terms with J_V in (5.6).

In our notation of eigenstates the gauge group structure is not explicit. Depending on the t channel gauge group state the above values are to be multiplied with a corresponding colour factor and with the sign factor $\xi_\pm(n) = 1 \pm (-1)^n$ in dependence of whether these gauge group representations are composed symmetrically or antisymmetrically in the adjoint colour indices. For the symmetric representations we have $\xi_+(\bar{n}+\nu+1)$ for the helicity states T,V and ξ_- for the helicity state A. The opposite sign factor applies for the antisymmetric representation.

The relation between the super multiplet states (5.7) and the parton states defined in (2.44) is obtained by doing the θ integrations in (5.7). In particular, $|\bar{n},0;f^*f\rangle, |\bar{n}-1,0;A^*,A\rangle, |\bar{n},0;f^*A\rangle, |\bar{n},0;A^*,f\rangle$ are linear combinations of the 4 supermultiplet states $|\bar{n},0,0;+,-\rangle$, $|\bar{n},1,0;+,-\rangle$, $|\bar{n},0,\frac{1}{2};+,-\rangle$, $|\bar{n}-1,1,\frac{1}{2};+,-\rangle$. i.e. 3 adjacent superconformal representations contribute to the conformal parton states of a given weight and different parton type (gluon, fermion). For this example we have to substitute for the lowest states

$\psi^{(0)}_{\bar n,\nu}$ from (4.11) and for the next-to-lowest states $\psi^{(\frac{1}{2})}_{\bar n,\nu}$ from (4.13) to obtain

$$\begin{aligned}
|\bar n, 0; 0; +, -\rangle &= \bar n |\bar n - 1, 0; A^* A\rangle + c^2 |\bar n, 0; f^* f\rangle, \\
|\bar n - 1, 1; \frac{1}{2}; +, -\rangle &= (\bar n + 3)|\bar n - 1, 0; A^* A\rangle - c^2 |\bar n, 0; f^* f\rangle, \\
|\bar n, 1; 0; +, -\rangle &= -c(|\bar n, 0; A^* f\rangle + |\bar n, 0; f^* A\rangle), \\
|\bar n, 0; \frac{1}{2}; +, -\rangle &= -c(\bar n + 2)(|\bar n, 0; A^* f\rangle - |\bar n, 0; f^* A\rangle\,.
\end{aligned} \tag{5.11}$$

The result implies analogous relations between the super multiplet eigenstates and eigenvalues of (5.6) and the eigenstates and eigenvalues of the particular gluon and fermion vertices, including the mixed gluon-fermion and annihilation-type interaction kernels, which can be written as matrix relations established already in [4]. Indeed, the matrix $\hat T$ of coefficients on the r.h.s. of (5.11) allows to express the anomalous dimension mixing matrix as

$$\hat T^{-1} \mathrm{diag}(\lambda_{\bar n,0}, \lambda_{\bar n-1,1}, \lambda_{\bar n,1}, \lambda_{\bar n,0}) \hat T\,. \tag{5.12}$$

For example, the element corresponding to the gluon-gluon parton splitting is obtained as

$$\gamma^g_g(\bar n - 1) = \frac{\bar n}{2\bar n + 3}\lambda_{\bar n,0} + \frac{\bar n + 3}{2\bar n + 3}\lambda_{\bar n-1,1}\,. \tag{5.13}$$

Substituting the above values for $\lambda_{\bar n,\nu}$ for the helicity configurations T, A, V (5.8), (5.9), (5.10) leads to the well known gluon one-loop anomalous dimensions up to the mentioned colour and sign factors.

6. Summary

Usually the renormalization of composite operators and the DGLAP/ERBL evolution of (generalized) parton distributions is formulated in terms of a basis of operators with their anomalous dimensions or as a set of evolution kernels in longitudinal momenta. Here we formulate instead quartic vertices of parton fields involving kernels in positions on the light ray, Fourier conjugate to the longitudinal momenta. This formulation allows to represent the (super) conformal symmetry of the one-loop parton interaction in the most explicit way. The kernels are (super) conformal 4-point functions. The effective parton interaction vertices provide a compact formulation of the leading twist exchange, where the twist equals to the number of exchanged partons.

This manifest symmetric representation can be obtained as a transformation of the conventional ones, e.g. by Fourier transformation of the momentum kernels. However it is not straightforward to recover the symmetric light-ray kernels in this way. The calculation following the effective action concept, starting from light cone form of the action, leads to the symmetric (one-loop) result via two standard loop integrals by a few transformations, combinig terms to remove spurious infrared divergencies and writing the vertices in terms of the conformal primary parton fields.

In the present paper we have applied this scheme [16] to $\mathcal{N} = 1$ super Yang-Mills theory emphasizing the close analogy to the gluodynamic ($\mathcal{N} = 0$) case. The light cone action has been written in a simple super field form involving one odd (θ) variable only. The calculation has been done in a form symmetric with respect to super translations (acting on θ and the light ray coordinate). The superconformal symmetry of the effective parton interaction has been formulated in the framework of the symmetry algebra $osp(2|1)$.

Parton super multiplet states are written in terms of wave functions on the super extended light ray (z, θ). They decompose into a polynomial basis corresponding to irreducible lowest weight representations of the symmetry algebra. There is a duality of the multi parton wave functions to composite operators of the parton fields by a symmetric scalar product.

The generic form of superconformal 4-point functions, being the kernels of symmetric two-parton operators, provides the guideline for identifying the wanted symmetric vertices. We find it convenient to start from symmetric closed-contour integral operators with kernels written in analytic power-like terms and to consider the kernels of integral operators on the light-ray as the result of the former by contour contraction.

Finally the effective vertices for $\mathcal{N} = 1$ super Yang Mills leading parton interaction are written in 3 terms involving particular superconformal kernels and contributing differently to the 3 different helicity states of the interacting parton pair.

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References

[1] V.G. Gribov and L.N. Lipatov, *Deep inelastic e p scattering in perturbation theory*, *Sov. J. Nucl. Phys.* **15** (1972) 438 [*Yad. Fiz.* **15** (1972) 781];
L.N. Lipatov, *Parton distribution functions in perturbation theory. (In russian)*, *Yad. Fiz.* **20** (1974) 532;
G. Altarelli and G. Parisi, *Asymptotic freedom in parton language*, *Nucl. Phys.* **B 126** (1977) 298;
Yu.L. Dokshitzer, *Zh. Eksp. Teor. Fiz.* **71** (1977) 1216.

[2] V.L. Chernyak and A.R. Zhitnitsky, *Asymptotic behavior of hadron form-factors in quark model. (In russian) Sov. Phys. JETP Lett.* **25** (1977) 510;
A.V. Efremov and A.V. Radyushkin, *Asymptotical behavior of pion electromagnetic form-factor in QCD*, *Theor. Math. Phys.* **42** (1980) 97; *Factorization and asymptotical behavior of pion form-factor in QCD*, *Phys. Lett.* **B 94** (1980) 245;
G.P. Lepage and S.J. Brodsky, *Exclusive processes in quantum chromodynamics: evolution equations for hadronic wave functions and the form-factors of mesons*, *Phys. Lett.* **B 87** (1979) 359; *Exclusive processes in perturbative quantum chromodynamics*, *Phys. Rev.* **D 22** (1980) 2157.

[3] Y.M. Makeenko, *Conformal operators in quantum chromodynamics*, *Sov. J. Nucl. Phys.* **33** (1981) 440;
T. Ohrndorf, *Constraints from conformal covariance on the mixing of operators of lowest twist*, *Nucl. Phys.* **B 198** (1982) 26.

[4] A.P. Bukhvostov, E.A. Kuraev and L.N. Lipatov, *Deep inelastic electron scattering by a polarized target in quantum chromodynamics*, *Zh. Eksp. Teor. Fiz.* **87** (1984) 37;
A.P. Bukhvostov, G.V. Frolov, L.N. Lipatov and E.A. Kuraev, *Evolution equations for quasi-partonic operators*, *Nucl. Phys.* **B 258** (1985) 601.

[5] D. Müller, *Conformal constraints and the evolution of the nonsinglet meson distribution amplitude*, *Phys. Rev.* **D 49** (1994) 2525; *The evolution of the pion distribution amplitude in next-to-leading-order*, *Phys. Rev.* **D 51** (1995) 3855 [hep-ph/9411338]; *Restricted conformal invariance in QCD and its predictive power for virtual two-photon processes*, *Phys. Rev.* **D 58** (1998) 054005 [hep-ph/9704406];
A.V. Belitsky and D. Müller, *Broken conformal invariance and spectrum of anomalous dimensions in QCD*, *Nucl. Phys.* **B 537** (1999) 397 [hep-ph/9804379].

[6] V.M. Braun, G.P. Korchemsky and D. Müller, *The uses of conformal symmetry in QCD*, *Prog. Part. Nucl. Phys.* **51** (2003) 311 [hep-ph/0306057].

[7] L.N. Lipatov, *Reggeization of the vector meson and the vacuum singularity in nonabelian gauge theories*, *Sov. J. Nucl. Phys.* **23** (1976) 338;
V.S. Fadin, E.A. Kuraev and L.N. Lipatov, *On the Pomeranchuk singularity in asymptotically free theories*, *Phys. Lett.* **B 60** (1975) 50; *Multi-reggeon processes in the Yang-Mills theory*, *Sov. Phys. JETP* **44** (1976) 443; *Multi-reggeon processes in the Yang-Mills theory*, *Sov. Phys. JETP* **45** (1977) 199; *The Pomeranchuk singularity in nonabelian gauge theories*, *Sov. Phys. JETP* **45** (1977) 199.

[8] L.N. Lipatov, *High-energy asymptotics of multicolor QCD and exactly solvable lattice models*, *Sov. Phys. JETP Lett.* **59** (1994) 596 [hep-th/9311037];
Sov. Phys. JETP Lett. **B342** (1994) 596.

[9] L.D. Faddeev and G.P. Korchemsky, *High-energy QCD as a completely integrable model*, *Phys. Lett.* **B 342** (1995) 311 [hep-th/9404173].

[10] H.J. De Vega and L.N. Lipatov, *Interaction of reggeized gluons in the Baxter-Sklyanin representation*, *Phys. Rev.* **D 64** (2001) 114019 [hep-ph/0107225].

[11] S.E. Derkachov, G.P. Korchemsky and A.N. Manashov, *Noncompact heisenberg spin magnets from high-energy QCD, I. Baxter Q-operator and separation of variables*, *Nucl. Phys.* **B 617** (2001) 375 [hep-th/0107193].

[12] V.M. Braun, S.E. Derkachov and A.N. Manashov, *Integrability of three-particle evolution equations in QCD*, *Phys. Rev. Lett.* **81** (1998) 2020 [hep-ph/9805225];
V.M. Braun, S.E. Derkachov, G.P. Korchemsky and A.N. Manashov, *Baryon distribution amplitudes in QCD*, *Nucl. Phys.* **B 553** (1999) 355 [hep-ph/9902375].

[13] A.V. Belitsky, *Fine structure of spectrum of twist-three operators in QCD*, *Phys. Lett.* **B 453** (1999) 59 [hep-ph/9902361]; *Integrability and wkb solution of twist-three evolution equations*, *Nucl. Phys.* **B 558** (1999) 259 [hep-ph/9903512]; *Renormalization of twist-three operators and integrable lattice models*, *Nucl. Phys.* **B 574** (2000) 407 [hep-ph/9907420].

[14] N. Beisert and M. Staudacher, *The $N = 4$ SYM integrable super spin chain*, *Nucl. Phys.* **B 670** (2003) 439 [hep-th/0307042].

[15] L. Dolan, C.R. Nappi and E. Witten, *A relation between approaches to integrability in superconformal Yang-Mills theory*, *J. High Energy Phys.* **10** (2003) 017 [hep-th/0308089].

[16] S.E. Derkachov and R. Kirschner, *Parton interactions in the bjorken limit of QCD*, *Phys. Rev.* **D 64** (2001) 074013 [hep-ph/0101174].

[17] A.V. Kotikov and L.N. Lipatov, *NLO corrections to the BFKL equation in QCD and in supersymmetric gauge theories*, *Nucl. Phys.* **B 582** (2000) 19 [hep-ph/0004008];
A.V. Kotikov and L.N. Lipatov, *DGLAP and BFKL equations in the $N = 4$ supersymmetric gauge theory*, *Nucl. Phys.* **B 661** (2003) 19 [hep-ph/0208220];
A.V. Kotikov, L.N. Lipatov and V.N. Velizhanin, *Anomalous dimensions of Wilson operators in $N = 4$ SYM theory*, *Phys. Lett.* **B 557** (2003) 114 [hep-ph/0301021].

[18] A.I. Onishchenko and V.N. Velizhanin, *Nonforward anomalous dimensions of Wilson operators in $N = 4$ super–Yang-Mills theory*, *J. High Energy Phys.* **02** (2004) 036 [hep-ph/0311329];
A.I. Onishchenko and V.N. Velizhanin, *Anomalous dimensions of twist-2 conformal operators in supersymmetric Wess-Zumino model*, hep-ph/0309222.

[19] A.V. Belitsky, S.E. Derkachov, G.P. Korchemsky and A.N. Manashov, *Superconformal operators in $N = 4$ super–Yang-Mills theory*, hep-th/0311104.

[20] A.V. Belitsky, S.E. Derkachov, G.P. Korchemsky and A.N. Manashov, *Quantum integrability in (super) Yang-Mills theory on the light-cone*, *Phys. Lett.* **B 594** (2004) 385 [hep-th/0403085].

[21] S. Mandelstam, *Covariant superspace with unconstrained fields*, *Phys. Lett.* **B 121** (1983) 30;
S. Mandelstam, *Light cone superspace and the ultraviolet finiteness of the $N = 4$ model*, *Nucl. Phys.* **B 213** (1983) 149.

[22] L. Brink, O. Lindgren and B.E.W. Nilsson, *The ultraviolet finiteness of the $N = 4$ Yang-Mills theory*, *Phys. Lett.* **B 123** (1983) 323;
$N = 4$ Yang-Mills theory on the light cone, *Nucl. Phys.* **B 212** (1983) 401.

[23] S.E. Derkachov and A.N. Manashov, *Generic scaling relation in the scalar Φ^4 model*, *J. Phys.* **A 29** (1996) 8011 [hep-th/9604173]; *The spectrum of the anomalous dimensions of the composite operators in epsilon expansion in the scalar Φ^4 field theory*, hep-th/9505110.